Solution of the inverse problem of magnetic induction tomography

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1 Introduction

Magnetic induction tomography (MIT) of biological tissue is used to reconstruct the changes $\Delta\kappa$ in the complex conductivity distribution $\kappa = \sigma + j\omega\varepsilon_0\varepsilon_r$ inside a target object [3]. MIT requires an array of excitation (EXC) and receiving (REC) coils. Each EXC couples an alternating magnetic field B_0 to the object under investigation. Changes $\Delta\kappa$ in a target region cause a field perturbation ΔB due to the induction of eddy currents in the object under investigation. The perturbation induces voltage changes ΔV in the receiver coils. The reconstruction of the absolute conductivity in a target region requires the solution of a complex inverse eddy current problem $\boldsymbol{\kappa} = \Psi^{-1}(\mathbf{V})$ which is ill-posed and usually underdetermined.

2 Methods

Newton-one-step reconstructor [1]:

$$\Delta\boldsymbol{\kappa} = \left(\mathbf{S}^T\mathbf{S} + \lambda\mathbf{R}^T\mathbf{R}\right)^{-1}\mathbf{S}^T\Delta\mathbf{V}$$

with **S** is called the sensitivity matrix and **R** is the regularization matrix.

2.1 Regularization

(1) $\mathbf{R}^T\mathbf{R} = \mathbf{E}$. Using the identity matrix is the most simple Tikhonov-regularization method.

(2) $\mathbf{R}^T\mathbf{R} = \mathbf{N}$ with **N** the neighbouring matrix. **N** is defined as:

$$N_{ij} = \begin{cases} n_n & i = j \\ -1 & i, j \quad \text{neighbours} \\ 0 & \text{otherwise} \end{cases}$$

n_n is the number of neighbouring elements for element i, whereby only elements with common faces are considered as neighbours.

(3) $\lambda\mathbf{R}^T\mathbf{R} = \mathbf{W}\mathbf{D}\mathbf{W}^T$ according to the variance uniformization approach [2] with $\mathbf{S} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{W}^T$ and $\mathbf{D} = \boldsymbol{\Sigma}/\mathbf{sqrt}(\mathbf{c}) - \boldsymbol{\Sigma}^2$ whereby **c** is a free scalar parameter.

(4) **Truncated singular value decomposition (TSVD):**

$$\Delta\boldsymbol{\kappa} = \mathbf{W}_t\boldsymbol{\Sigma}_t^{-1}\mathbf{U}_t^T\Delta\mathbf{V}$$

whereby t denotes the truncation level of the original matrices **W**, **Σ** and **U**, thus removing the contributions of singular values with index > t.

2.2 Modeling setup

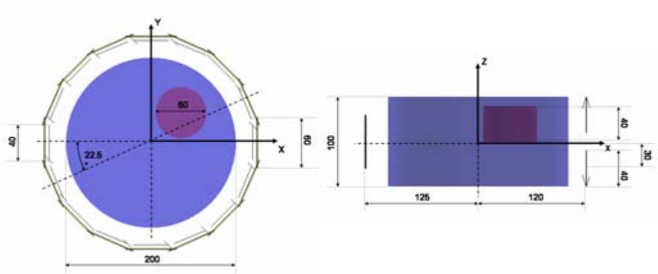

Fig. 1: Modelling setup: (left) Top view, (right) front view, all measurements in mm

The measured data were simulated when changing the conductivity of the cylindrical perturbation from 1 S/m (= background conductivity) to 2 S/m. The permittivity was kept constant with 80 in both cylinders. The excitation frequency was 700 kHz. 1 % uncorrelated Gaussian noise was added to the voltage data in order to simulate the noise of the receiver channels.

3 Results

Fig. 2 shows the reconstructed images for the four different regularization methods. All four methods performed comparatively well in localizing the disturbed object.

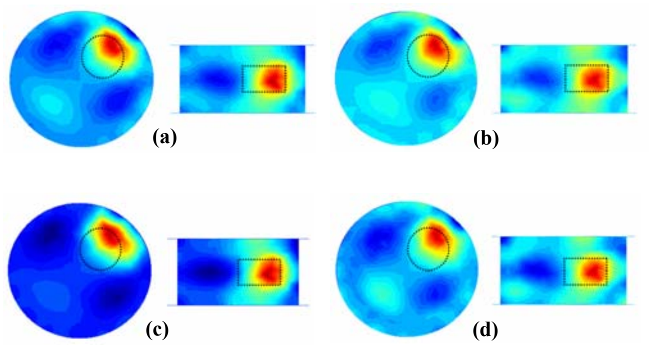

Fig. 2: Top view and front view of the reconstructed real part of $\Delta\kappa$, regularization method: (a) identity matrix, (b) neighbouring matrix, (c) variance uniformization, (d) truncated SVD

4 Conclusions

The shown results demonstrate the feasibility of image reconstruction from MIT-data with the same methods as suggested for EIT. This finding is not self-evident, as the sensitivity distribution is significantly different in EIT and MIT. When comparing the regularization schemes, the variance uniformization approach yield the smoothest solutions, as expected. The most homogeneous background is obtained also with variance uniformization, according to the variance constraint based on this regularization scheme.

5 Acknowledgements

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6 References

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